

**BIRZEIT UNIVERSITY**  
**MATHEMATICS DEPARTMENT**  
**Stat 236**

Summer semester 2014/2015- Midterm Exam

35

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$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

Z - Score:  $z = \frac{x - \mu}{\sigma}$

Correlation coefficient:  $r = \frac{s_{xy}}{s_x s_y} = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$

Covariance:  $s_{xy} = \frac{\sum (x - \bar{x})(y - \bar{y})}{n-1}$

Permutations:  $nPr = \frac{n!}{(n-r)!}$

Combinations:  $nCr = \frac{n!}{(n-r)!r!}$

Conditional probability:  $p(A \setminus B) = \frac{p(A \cap B)}{p(B)}$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

Discrete Random Variable  $E(X) = \mu = \sum xf(x)$

$Var(X) = \sum (x - \mu)^2 f(x)$

Uniform:

$E = \frac{a+b}{2}$

$Var. = \frac{(b-a)^2}{12}$

Binomial Probability Distribution  $P(X=x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$E(X) = np, \sigma(X) = \sqrt{np(1-p)}$

Poisson Probability :  $f(x) = \frac{\mu^x e^{-\mu}}{x!}$

Exponential Probability Distribution

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

Question # 1: (22 points) Circle the correct answer.

- Student's **university numbers** consist of numeric values. Therefore, student's number is an example of
  - Quantitative data.
  - Qualitative data
- The number Stat Quizzes in this semester is an example of what scale :
  - Ordinal
  - Nominal
  - Interval
  - Ratio
- which one of the following is a measure of Asymmetry:
  - The median
  - The skewness
  - The IQR
  - The covariance.
- Which one of the following graphical presentation is appropriate for qualitative data:
  - Histogram
  - Ogive
  - pie chart
  - cumulative frequency table
- To study the relation between gender (male, female) and the Tawjihi average the appropriate summary is?
  - Histogram.
  - Bar graph.
  - Cross tabulation.
  - Scatter diagram.
- If the frequency of a Business students is 100 from 720 Commerce students then its angle in the **pie chart** is:
  - 25
  - 50
  - 100
  - 200
$$\frac{100 \times 360}{720}$$
- If A, B are two independent events, where  $P(A) = 0.4$ ,  $P(A \cup B) = 0.7$ , then  $P(B) =$ 
  - 0.3
  - 0.4
  - 0.5
  - Can't be determined by the given informations.
$$P(A \cup B) = P(A) + P(B)$$

$$0.7 = 0.4 + P(B)$$
- How many **3-digit numbers** can be formed from the digits 2, 3, 4, 5, and 6, without repeating the digits??
  - 3
  - 10
  - 20
  - 60
$$n! / (n - r)!$$

$$\frac{5!}{3! (2!) = \frac{120}{6 \cdot 2}$$
- An experiment consists of selecting a student body president and vice president. All undergraduate students (first year through fourth year) are eligible for these offices. How many sample points (possible outcomes as to the classifications) exist?
  - 8
  - 16
  - 32
  - 64
$$2^4$$

10. The random variable x is known to be uniformly distributed between 50 and 90.

Find  $P_{80} =$

- 70
- 78
- 80
- 82

$$\frac{90 - 80}{90 - 50}$$

$$\frac{1}{b-a} = \frac{1}{90-50}$$

0.8

$$.7851 = \frac{90 - x}{40}$$

$$.8 = \frac{x - 90}{40}$$

0.84

31.538

11. Events that have no sample points in common are

- a. independent events.
- b. mutually exclusive events.
- c. complement events.
- d. simple events.

9915

12. in which distribution the data values have the same frequency:

- a. Bimodal distribution
- b. Uniform distribution
- c. Poisson distribution.
- d. Exponential distribution.

$1 - P(40)$

13. Suppose that X is a binomial random variable with 100 trials and probability of success equal 0.3. Find the approximated probability of at least forty successes

- a. 0.9808
- b. 0.011
- c. 0.0192
- d. 0.0146

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$\binom{100}{40} (0.3)^{40} (0.7)^{60} =$$

CHF

~~$P(X \geq 40)$~~

$P \geq 40$

(14-15) Given the probability distribution of the number of family members

Number of family members (x)	2	3	4	5	6	7 or more
Probability $P(X = x) = f(x)$	0.03	0.14	0.39	0.26	0.1	0.08

14. Find  $P(X < 4)$ .

- a. 0.17
- b. 0.44
- c. 0.56
- d. 0.83

x	P(x)
2	0.03
3	0.14
4	0.39
5	0.26
6	0.1
7	0.08

15. Find  $E(X) =$

- a. 4
- b. 4.5
- c. 5.5
- d. 6

(16-18) The average life for a TV set is 10 years. Suppose that the lifetime follows an exponential distribution.

16. Find The probability that the life time is between 3 and 6 years is

- a. 0.153
- b. 0.192
- c. 0.741
- d. 0.808

$3 \leq x \leq 6$

$\rightarrow e^{-\frac{3}{10}} - e^{-\frac{6}{10}}$

$.74081822 -$

17. Find The probability that the life time is more than 5 years is

- a. 0.6065
- b. 0.6487
- c. 0.3935
- d. None of the above

$x > 5$

$e^{-\frac{5}{10}}$

$\rightarrow e^{-\frac{5}{10}}$

$67 \rightarrow 100\%$   
 $2 \rightarrow 75\%$   
 $75 = 100\%$

$e^{-\frac{2.8}{10}}$   
 $\rightarrow e^{-\frac{2.8}{10}}$   
 $= 2.718$

$$0.75 = e^{-\frac{a}{m}} \quad \boxed{0.52}$$

18. 75% of the TV life will be at most how long?

- a. 2.8 years
- b. 13.8 years
- c. 32.1 years
- d. 43.2 years

$$G=4 \quad m=20$$

(19-21) AlBireh Arab Hospital has noted that they admit an average of 4 customers arrive at the bank every 20 min. Define the random variable X to be the number of admitted customers

19. What is the appropriate probability distribution for X?

- a. Poisson
- b. Binomial
- c. Uniform
- d. Exponential

$$\frac{40-20}{4} = 5$$

20. Find the probability that 3 customers arrive in 40 min

- a. 0.0286
- b. 0.1804
- c. 0.1954
- d. 0.8053

$$\frac{2^3 \cdot e^{-2}}{3!} \quad \frac{4^3 \cdot e^{-4}}{3!}$$

$$\frac{4^3 \cdot e^{-4}}{3!} \quad \frac{5^3 \cdot e^{-5}}{3!}$$

21. Find the probability that more than 2 customers arrive 20 min

- a. 0.0915
- b. 0.2381
- c. 0.7619
- d. 0.9085

$$P(X > 2)$$

$$\frac{1^2 \cdot e^{-1}}{2!}$$

22. A basketball player has probability 0.8 of making a free shot. What is the probability that he makes exactly 3 if he throws 6 shots from the line?

- a. 0.0819
- b. 0.8202
- c. 0.9131
- d. 0.0381

تجارت

$$\binom{6}{3} \cdot 0.8^3 \cdot 0.2^3$$

20

$$\binom{6}{3}$$

$$P(X=2)$$

$$P(3) = P(4)$$

$$0.061$$

Handwritten calculations and scribbles at the bottom of the page, including:

- $1 - (P(0) + P(1) + P(2))$
- $\frac{2^0 \cdot e^{-2}}{0!} + \frac{2^1 \cdot e^{-2}}{1!} + \frac{2^2 \cdot e^{-2}}{2!}$
- $\frac{2^3 \cdot e^{-2}}{3!}$
- $\frac{4^3 \cdot e^{-4}}{3!}$
- $\frac{2^3 \cdot e^{-2}}{3!}$

**Question # 2 (8 points)**

The following data show the number of hours worked by two workers x, y in 8 days. answer the following questions.

x	y	$(y-\bar{y})$	$(y-\bar{y})^2$	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$
2	5	-6	36	-5	25	30
4	7	-4	16	-3	9	12
8	9	-2	4	1	1	-2
5	12	1	1	-2	4	-2
7	10	-1	1	0	0	0
8	13	2	4	1	1	2
16	16	5	25	9	81	45
6	16	5	25	-1	1	-5
			112		122	30

1. Compute  $\bar{x}, \bar{y}$

$$\bar{x} = \frac{2+4+8+5+7+8+16+6}{8} = 7$$

$$\bar{y} = \frac{5+7+9+12+10+13+16+16}{8} = 11$$

2. Compute and interpret the sample correlation coefficient.

$$r = \frac{S_{xy}}{S_x \cdot S_y}$$

$$S_{xy} = \frac{\sum (x-\bar{x})(y-\bar{y})}{n-1} = \frac{30}{8-1} = 4.2857$$

$$S_x = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{122}{8-1}} = 4.1747$$

$$S_y = \sqrt{\frac{\sum (y-\bar{y})^2}{n-1}} = \sqrt{\frac{112}{8-1}} = 4$$

$$r = \frac{4.2857}{(4.1747)(4)} = 0.68439$$

Correlation coefficient = 0.68439

↳ the relation is

Positive Strong  
relation.

**Question # 3 (8 points)**

The distribution of scores for a particular exam follows a normal distribution with mean of 70 and standard deviation of 6. Answer the following:

- Find the probability that a randomly selected student get more than 74.5 in the exam?
- Find the probability that a randomly selected student get between 68.5 and 70 in the exam?
- If the least 8% will fail the course, what is the Score should be obtained to avoid Failing the course?
- If you got a score of 79 in this class, at what percentile is your score?

عل درجہ

بہتر

$$M = 70 \quad \sigma = 6$$

$$Z = \frac{x - M}{\sigma}$$

c) - 0.08 Fail

$$1 - 0.08 = .92$$

from table →

$$\frac{.9192 + .9207}{2} = 0.91995$$

a) -  $P(x > 74.5)$

$$Z = \frac{74.5 - 70}{6} = 0.75$$

from table → .67

The probability is →  $1 - .67 = .33$

$$Z = \frac{x - M}{\sigma}$$

$$0.91995 = \frac{x - 70}{6}$$

$$5.5197 = x - 70$$

$$x = 75.5197$$

b) -  $P(68.5 < x < 70)$

$$Z = \frac{68.5 - 70}{6} = -.25$$

$$Z = \frac{70 - 70}{6} = 0$$

↳  $P(-.25 < x < 0)$

↳  $P(x < 0) - P(x < -.25)$

↳  $.50 - (1 - .5987)$

↳  $.5 - .4013 = 0.0987$

d) - a) Find index.

$$\frac{79}{100} \times 100 = 79\%$$

3

**Question # 4 (6 points)**

The STAT grades of 50 students in the first and their genders are shown below. A student is randomly selected from these 50

Gender	STAT Grade			Total
	Less than 20	20 up to 30	30 and more	
Female	4	16	12	32
Male	6	2	10	18
Total	10	18	22	50

Total = 50 students  
relative to

1. What is the probability that this student is male and his grade 20 up to 30?

Probability  $\rightarrow \frac{2}{50} = 0.04$

2. What is the probability that this student is female?

Female  $\rightarrow \frac{32}{50} = 0.64$

3. What is the probability that this student has grade 20 and more?

$\frac{16+12+2+10}{50} = 0.8$

4. What is the probability that this student has grade less than 10 or female?

~~P < 10~~  
P < 10

$\rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4}{32} = 0.125$

5. If this student has grade 30 and more, what is the probability that he is male?

$\frac{P(A \cap B)}{P(B)} = \frac{10}{18} = 0.556$

6. If this student is female, what is the probability that her grade 20 up to 30?

$P(\text{female} / \text{grade } 20 \text{ up } 30)$

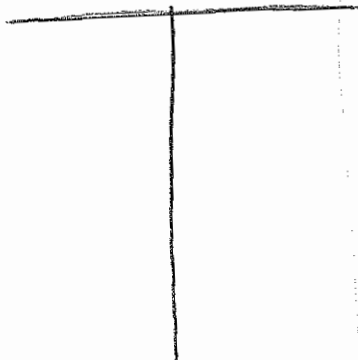
$= \frac{P(F \cap G_{20 \text{ up } 30})}{P(G_{20 \text{ up } 30})} = \frac{16}{18} = 0.8889$

**Question # 5 (2 points)**

The random variable X is known to be uniformly distributed where  $E(x) = 5$ ,  $\text{var}(x) = \frac{1}{3}$ .

Write the probability density function. PDF

A and B



$$a+b = 10 \rightarrow (1)$$

$$-a+b = 2 \rightarrow (2)$$


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$$2b = 12$$

$$\boxed{b = 6}$$

$$\boxed{a = 4}$$

$$E(x) = \frac{a+b}{2}$$

$$\rightarrow 5 = \frac{a+b}{2} \Rightarrow a+b = 10 \rightarrow (1)$$

$$\text{var} = \frac{(b-a)^2}{12}$$

$$\frac{1}{3} = \frac{(b-a)^2}{12}$$

$$12 = 3(b-a)^2$$

$$\sqrt{4} = (b-a)^2$$

$$2 = b-a \rightarrow (2)$$

**Question # 6 (4 points)**

Stat Sections in this summer are 6, the probability that a class don't cover all stat material is 10%. Find the probability that

- Two sections will cover the material.  $P = 2$
- At least one section will not cover stat material

$$a) - \binom{6}{2} (0.9)^2 \cdot (0.1)^4$$

$$= (15) (0.81) \cdot (0.0001)$$

$$= 1.215 \times 10^{-3}$$

b) -  $1 - P(0)$

$$\rightarrow \binom{6}{0} (0.1)^0 (0.9)^6$$

$$= (1) (1) (0.531441) = 0.531441$$

$$1 - 0.531441$$

$$= \boxed{0.468559}$$

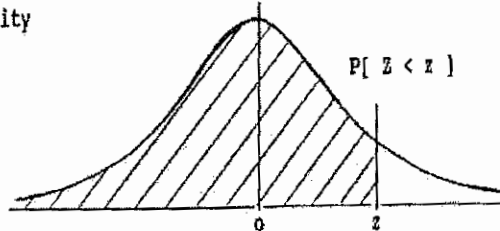


STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value  $z$  i.e.

$$P\{Z < z\} = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) dz$$



$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
$z$	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
$P$	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000